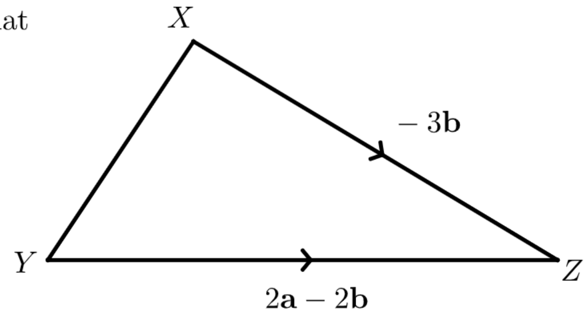
Question 1

Q is the point on XY such that
$XQ : QY = 2 : 1$

Find the vector $\overrightarrow{ZQ}$
in terms of $\mathbf{a}$ and $\mathbf{b}$.



Question 2

n is a positive integer.
Show that $3n^2 - 9 + (n - 6)^2$ is always odd.

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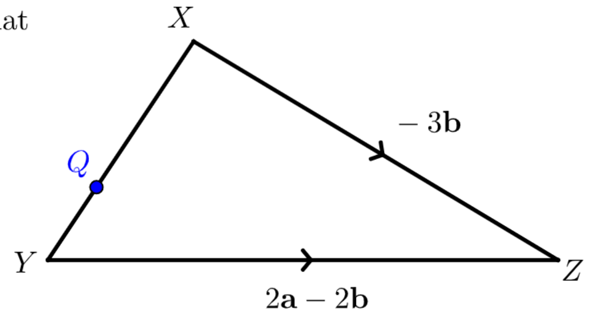

First note that $\overrightarrow{XQ} = \frac{2}{3}\overrightarrow{XY}$.

$$\overrightarrow{XY} = \overrightarrow{XZ} + \overrightarrow{ZY}$$
$$= \overrightarrow{XZ} + \left(-\overrightarrow{YZ}\right) = (-3\mathbf{b}) + (-2\mathbf{a} + 2\mathbf{b}) = -2\mathbf{a} - \mathbf{b}$$

So $\overrightarrow{XQ} = \frac{2}{3}(-2\mathbf{a} - \mathbf{b}) = -\frac{4}{3}\mathbf{a} - \frac{2}{3}\mathbf{b}$

Now, $\overrightarrow{ZQ} = \overrightarrow{ZX} + \overrightarrow{XQ}$

$$= -\overrightarrow{XZ} + \overrightarrow{XQ} = (3\mathbf{b}) + \left(-\frac{4}{3}\mathbf{a} - \frac{2}{3}\mathbf{b}\right) = -\frac{4}{3}\mathbf{a} + \frac{7}{3}\mathbf{b}$$

Question 2

n is a positive integer.
Show that $3n^2 - 9 + (n-6)^2$ is always odd.

Expanding and simplifying, we get $4n^2 - 12n + 27$
We can write this as $2(2n^2 - 6n + 13) + 1$
This is always odd.